Non linear behavior of electrostatically actuated micro-structures

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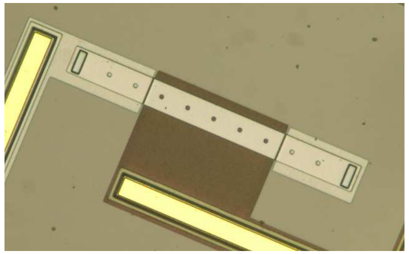
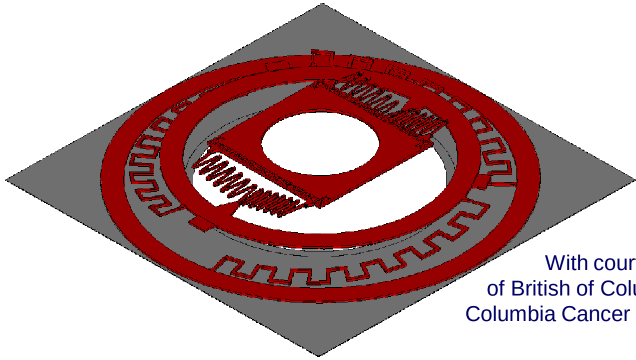
Outline

- Introduction
- Basic principles
- Finite element formulation
- Nonlinear algorithms
- Validation & examples
- Oofelie::MEMS, driven by SAMCEF Field
- Perspectives

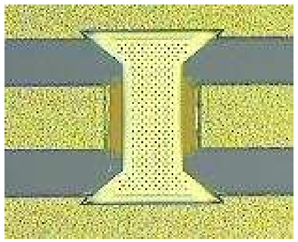
Introduction

- Electrostatics is often used in micro-systems
  - RF Switches
  - Micro-resonators (gyrometers,...)
  - Micro-lens for biomedical application
  - Adaptative optics
  - ...

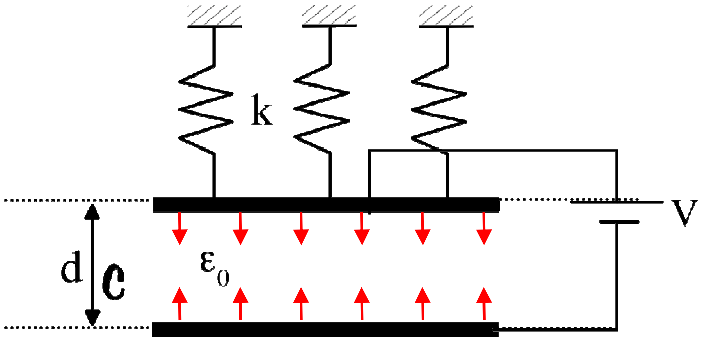




With courtesy of University of British of Columbia and British Columbia Cancer Research Centre

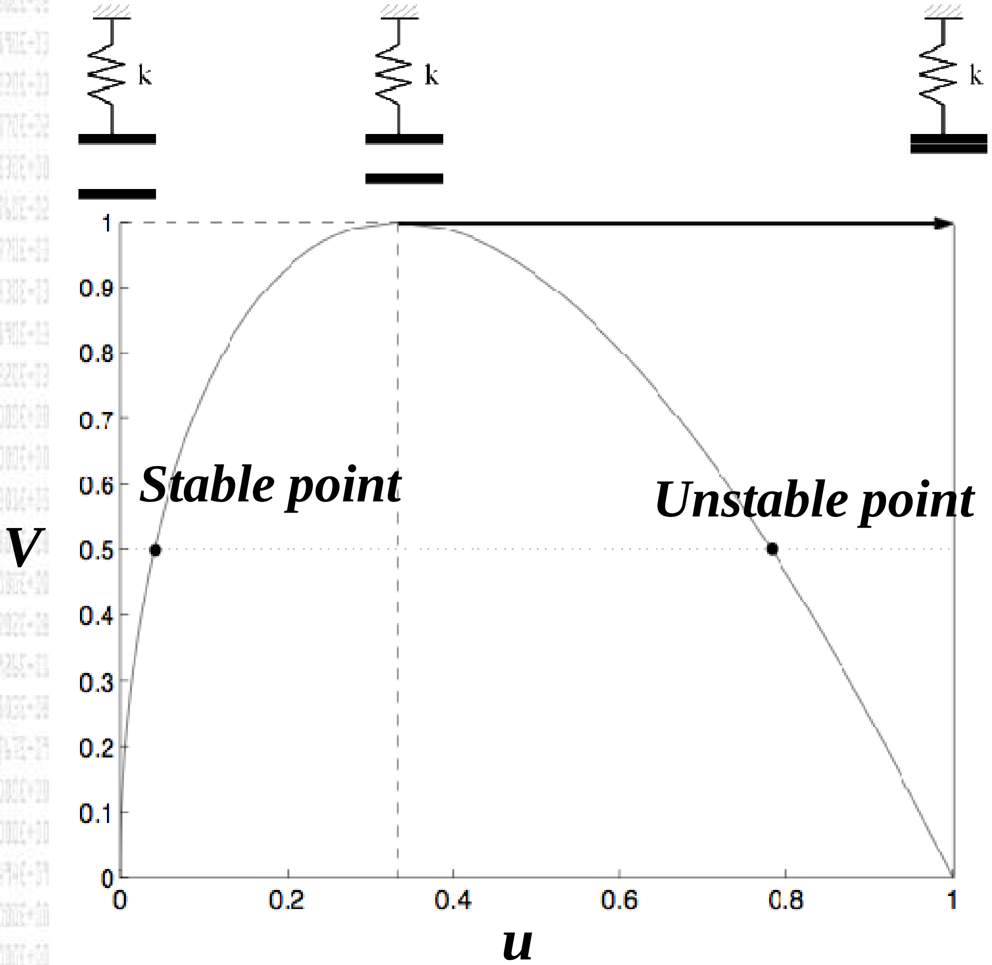


- 1D Reference problem



Basic Principles: Static analysis

- Static equilibrium equation: $k(d - d_0) = -\frac{1}{2}\varepsilon_0 \frac{V^2}{d^2}$



Static Pull-in Voltage:

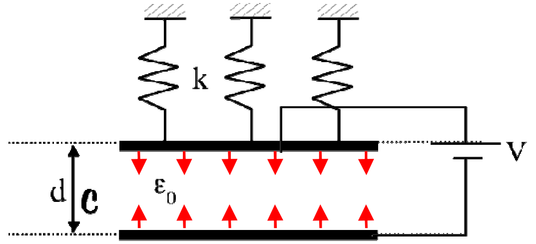
The voltage amplitude for which only one equilibrium position exists

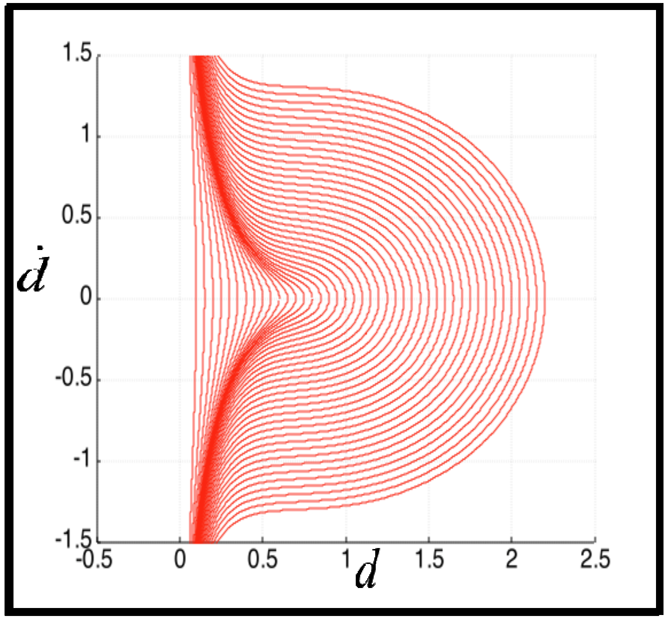
$$V_{spi} = \sqrt{\frac{8}{27}\frac{kd_0^3}{\varepsilon_0}}$$

Electromechanical Problem: Dynamic Analysis

- **Dynamic Equation:** $$m\ddot{d} = -k(d - d_0) - \frac{1}{2}\varepsilon_0 \frac{V^2}{d^2}$$

  - ***V = 0, the phase diagram is an ellipsoid***
  - ***V = V*, an unstable zone appears***
  - ***V ↗, the stability zone is reduced and disappears when the static pull-in voltage is reached***





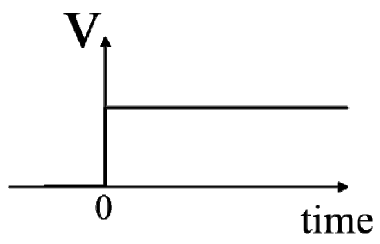
➡ 2 new parameters

Basic principles: Dynamic Pull-In Voltage

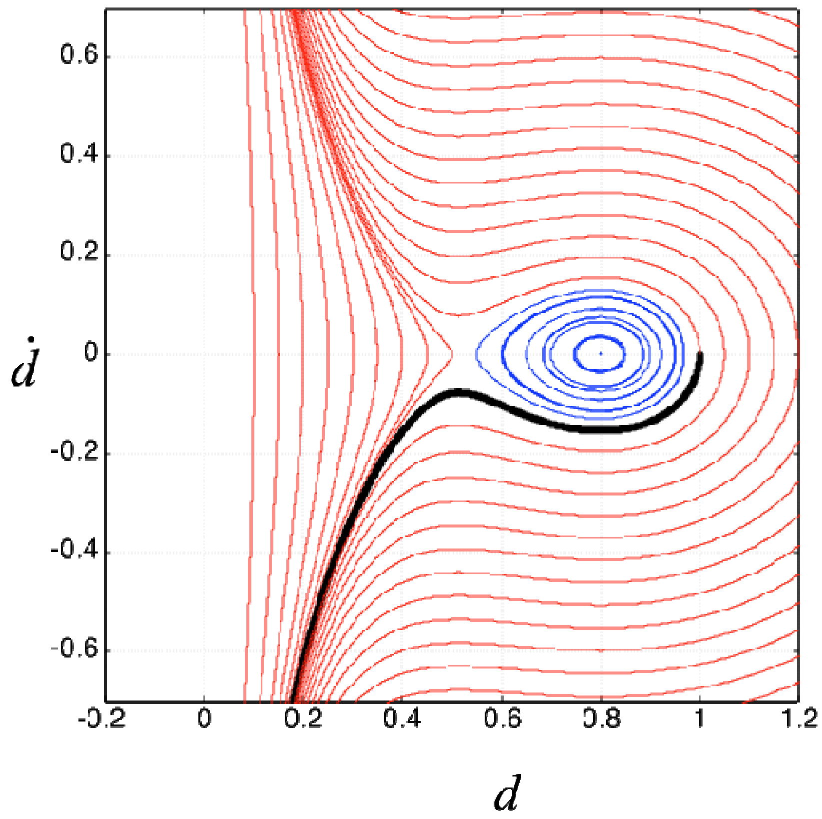
Dynamic Pull-in Voltage:

When the voltage is applied suddenly, the system becomes unstable

Step of voltage



$$\boxed{V_{dpi} = 0.5\sqrt{\frac{k d_0^3}{\varepsilon_0}}}$$



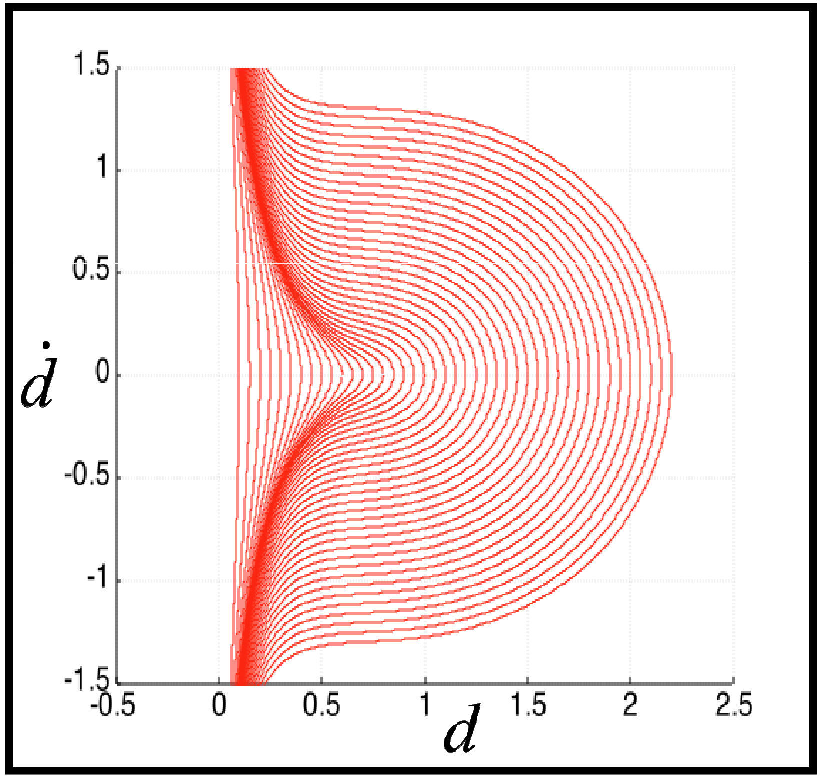
Basic principles: Dynamic Pull-in time

Dynamic Pull-in Time:

The time needed for the plates to stick together when the static pull-in voltage is applied

$$\boxed{t_{dpi} = 4.35\sqrt{\frac{m}{k}}}$$

Independent of the gap and the permittivity



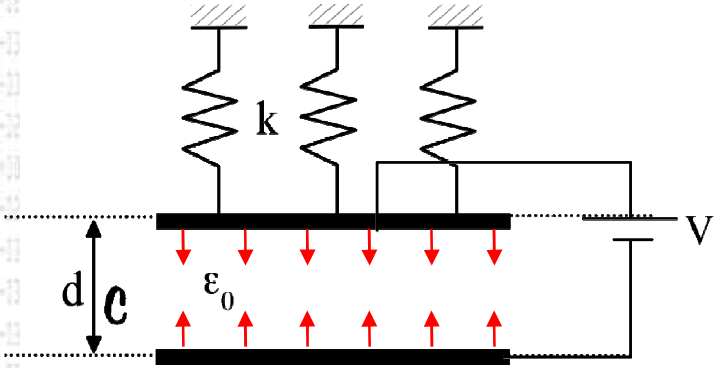
Finite Element: Strongly Coupled Formulation

- Analytical expression of the tangent stiffness matrix
- Advantages
  - Faster convergence to the non-linear solutions
  - Accurate evaluation of the pull-in voltage
  - Modal analysis
  - Time integration
- Full explanation for 1D formulation
- 2D and 3D extension is "obvious" using "Virtual Work" principle

1D Formulation

Gibb's energy density:

$$G = \underbrace{\frac{1}{2}\mathbf{S}^T\mathbf{T}}_{W_m} - \underbrace{\frac{1}{2}\mathbf{D}^T\mathbf{E}}_{W_e}$$



with

$$W_m = \frac{1}{2}k(d-d_0)^2 = \frac{1}{2}ku^2$$

$$W_e = \frac{1}{2}\int_{C(u)} \frac{V}{d}\varepsilon_0\frac{V}{d}dx$$

- **Mechanical force:** $$f_m = \frac{\partial W_m}{\partial u} = ku$$

- **Electrical charge:** $$q_e = \frac{\partial W_e}{\partial V} = \varepsilon_0\frac{V}{d}$$

1D Formulation

- ***Electrostatic force:***

$$f_e = -\frac{\partial W_e}{\partial u} = -\lim_{\delta \to 0} \frac{W_e^* - W_e}{\delta}$$

with

$$W_e = \frac{1}{2}\int_0^d \varepsilon_0 \frac{\partial V}{\partial x}\frac{\partial V}{\partial x}dx = \frac{1}{2}\varepsilon_0 \frac{V^2}{d}$$

$$W_e^* = \frac{1}{2}\int_0^{d+\delta} \varepsilon_0 \frac{\partial V}{\partial x}\frac{\partial V}{\partial x}dx$$

$$= \frac{1}{2}\int_0^d \varepsilon_0 \frac{d}{d+\delta}\frac{\partial V}{\partial \xi}\frac{d}{d+\delta}\frac{\partial V}{\partial \xi}\frac{d+\delta}{d}d\xi$$

$$= \frac{1}{2}\frac{d}{d+\delta}\int_0^d \varepsilon_0 \frac{\partial V}{\partial \xi}\frac{\partial V}{\partial \xi}d\xi = \frac{1}{2}\varepsilon_0 \frac{d}{d+\delta}\frac{V^2}{d}$$

$$\longrightarrow \quad f_e = \frac{1}{2}\varepsilon_0 \frac{V^2}{d^2}$$

1D Formulation

- **Equilibrium Equations**

$$\begin{cases} ku + f_e = f_{ext} \\ -q_e = -q_{ext} \end{cases}$$

$$f_e = \frac{\varepsilon_0 V^2}{d^2}$$

$$q_e = \frac{\varepsilon_0 V}{d}$$

- **Linearisation around a position** $(\tilde{u}, \tilde{V})$

$$\begin{cases} k(\tilde{u} + du) + f_e(u,V) = f_{ext} + df_{ext} \\ -q_e(u,V) = -q_{ext} - dq_{ext} \end{cases}$$

  - Electric force $\quad f_e(u,V) = f_e(\tilde{u},\tilde{V}) - \dfrac{\varepsilon_0 \tilde{V}^2}{\tilde{d}^3} du + \dfrac{\varepsilon_0 \tilde{V}}{\tilde{d}^2} dV$
  - Electric charge $\quad q_e(u,V) = q_e(\tilde{u},\tilde{V}) - \dfrac{\varepsilon_0 \tilde{V}}{\tilde{d}^2} du + \dfrac{\varepsilon_0}{\tilde{d}} dV$

- **Tangent stiffness Matrix**

$$\begin{pmatrix} k - \frac{\varepsilon_0 \tilde{V}^2}{\tilde{d}^3} & \frac{\varepsilon_0 \tilde{V}}{\tilde{d}^2} \\ \frac{\varepsilon_0 \tilde{V}}{\tilde{d}^2} & -\frac{\varepsilon_0}{\tilde{d}} \end{pmatrix} \begin{pmatrix} du \\ dV \end{pmatrix} = \begin{pmatrix} df_{ext} \\ -dq_{ext} \end{pmatrix}$$

General FE Formulation (2D and 3D)

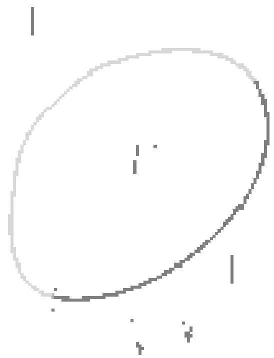
- ***Gibbs energy density***
- ***On a volume***

$$G = \frac{1}{2}\mathbf{S}^{\mathbf{T}}\mathbf{T} - \frac{1}{2}\mathbf{D}^{\mathbf{T}}\mathbf{E}$$

$$W_{int} = \frac{1}{2}\int_V \boldsymbol{S}^T \boldsymbol{T}\, dV - \frac{1}{2}\int_{V(u)} \boldsymbol{D}^T \boldsymbol{E}\, dV = W_m - W_e$$

Mechanical domain

Electric domain



$(\mathbf{u}, \phi)$

Unknowns

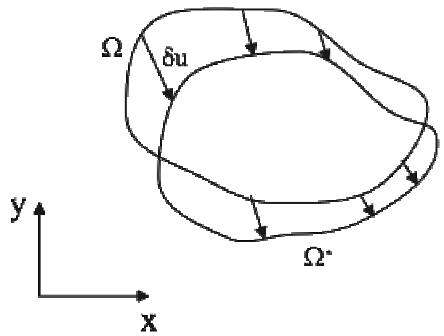
$$\begin{cases} S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) & \text{in } V \\ \boldsymbol{u} = \overline{\boldsymbol{u}} & \text{on } \Gamma_u \end{cases}$$

$$\begin{cases} \boldsymbol{E} = -\nabla\phi & \text{in } V \\ \phi = \overline{\phi} & \text{on } \Gamma_\phi \end{cases}$$

General FE Formulation (2D & 3D)

- **_Virtual work_**

$$\begin{cases} \boldsymbol{u}^* = \boldsymbol{u} - \delta \boldsymbol{u} \\ \phi^* = \phi + \delta\phi \end{cases}$$



- **_Equilibrium equations_**

Pure Mech.

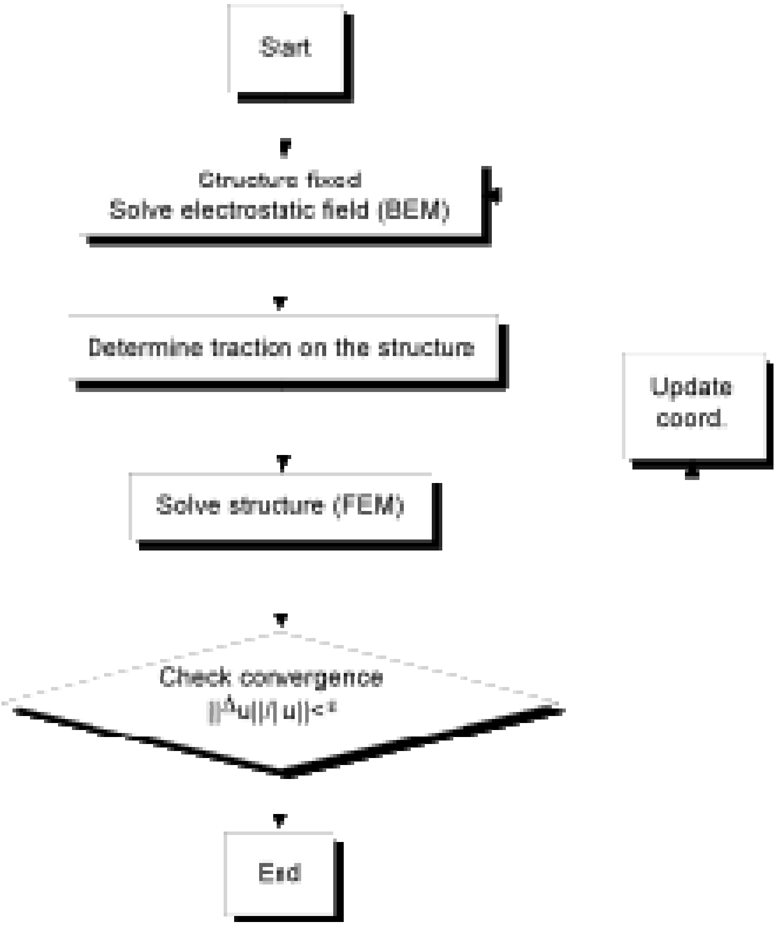
Electrostatic force

$$\begin{cases} \boldsymbol{f}_m \cdot \delta \boldsymbol{u} = \delta_u W_{int} = \delta_u W_m - \delta_u W_e \\ f_q \, \delta\phi = \delta_\phi W_{int} = \underbrace{\delta_\phi W_m}_{=0} - \delta_\phi W_e \end{cases}$$

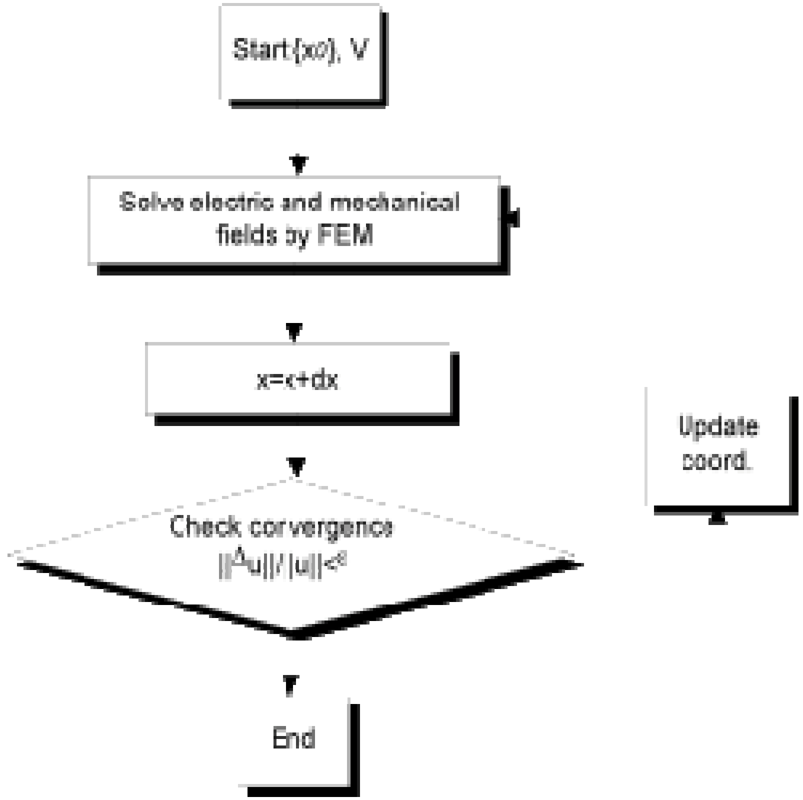
Pure Electric

Nonlinear solvers: Staggered vs Monolithical

Staggered Method

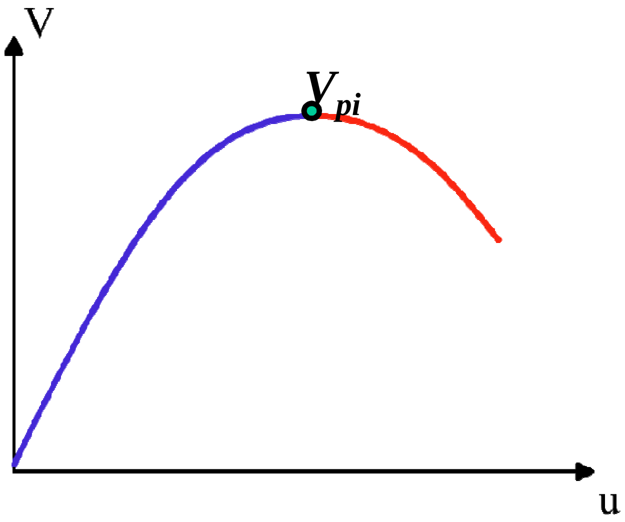


Monolithical Method

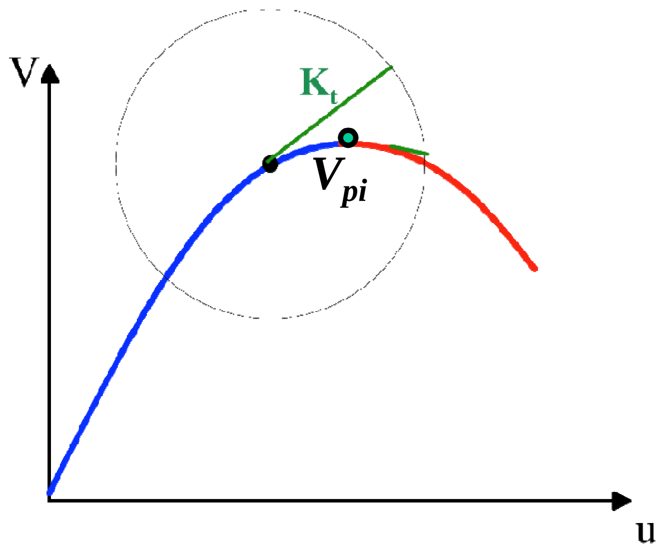


Nonlinear solvers for Pull-In computation

Newton Raphson



Riks-Crisfield



Nonlinear solver: Modal analysis

In FEM formulation: $$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}_e(\boldsymbol{\varphi}(\mathbf{u}))$$

1st method: Projection on the first mech. eigenmode

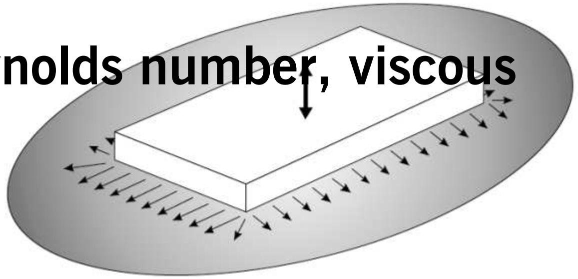
Natural frequency $$\lambda \simeq \frac{k - \frac{\partial f_e}{\partial q}}{m}$$

2nd method: Linearisation around an equilibrium position

$$\Rightarrow \quad (\mathbf{K}_{\tan}(\mathbf{q}_0) - \omega^2 \mathbf{M})\Delta\mathbf{q} = \mathbf{0}$$

Integration of fluid damping (in film)

- **Assumptions:**
  - laminar and fully developed flow (low Reynolds number, viscous dominated flow)
  - pressure does not vary in z-direction
  - the fluid does not slip at the walls
  - very low Knudsen number ($Kn = \lambda / h_0$, where λ mean free path of molecules, inversely proportional to P).

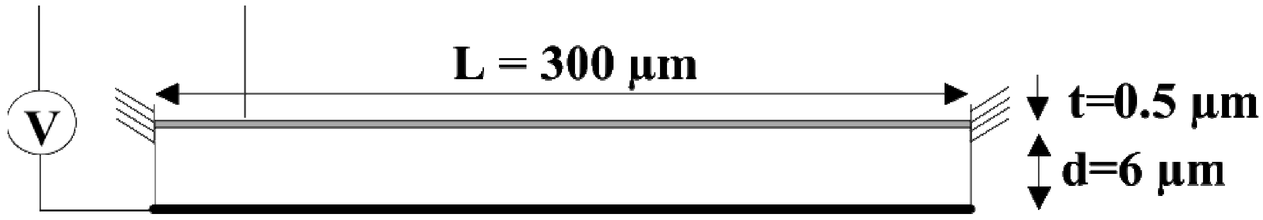


- **Nonlinear Reynolds equation:**

$$\frac{\partial}{\partial x}\left(\frac{Ph^3}{\mu}\frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{Ph^3}{\mu}\frac{\partial P}{\partial y}\right) = 12\frac{\partial (Ph)}{\partial t}$$

- **Last assumption for linear damping:**
  - small amplitude motion in comparison with h_0

Application 1: Thin Beam, large gap

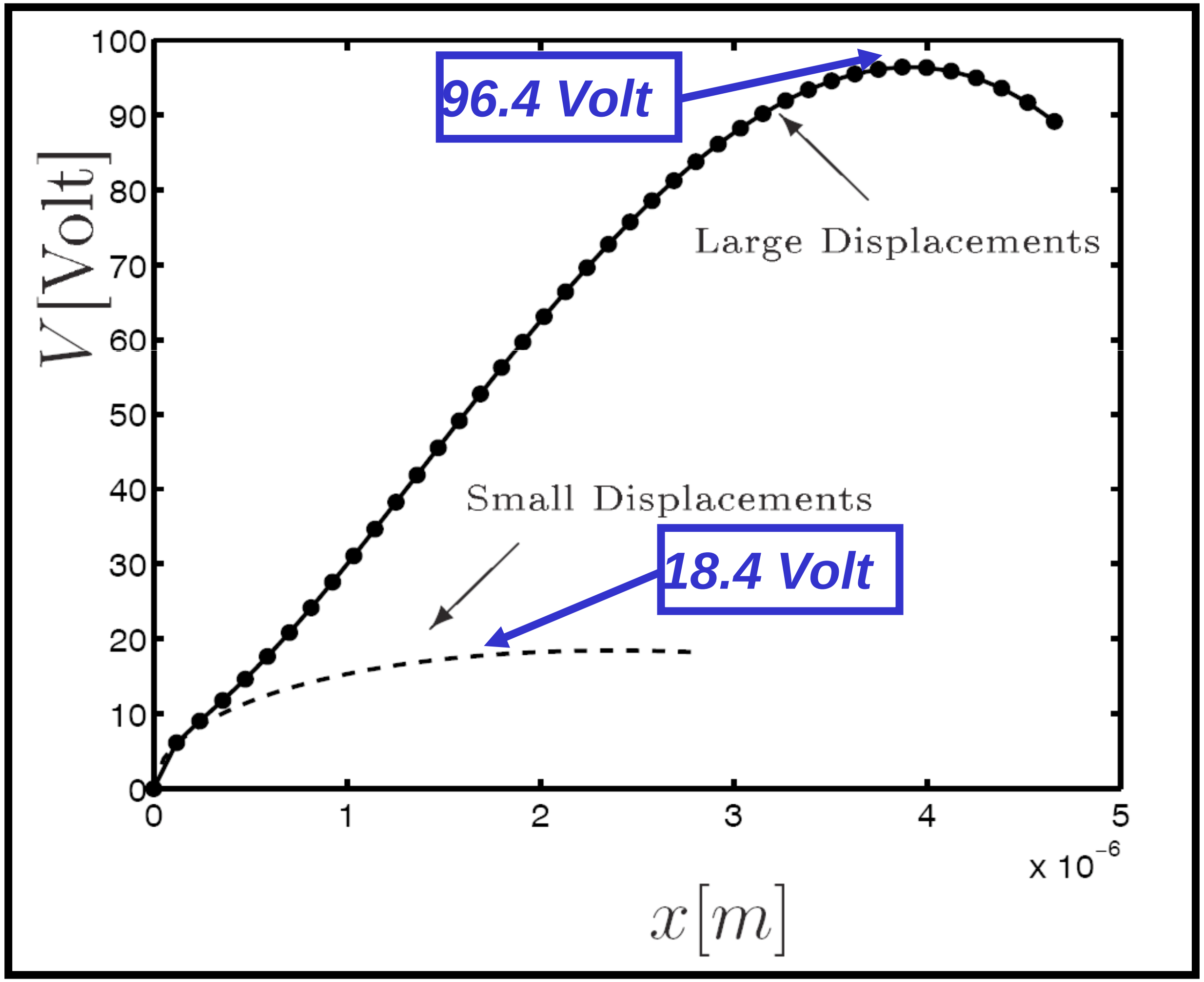


The stiffness of the beam becomes non-linear with respect to displacement (cable effect)

➥ ***Large displacements FE formulation taking into account geometric NL was used***

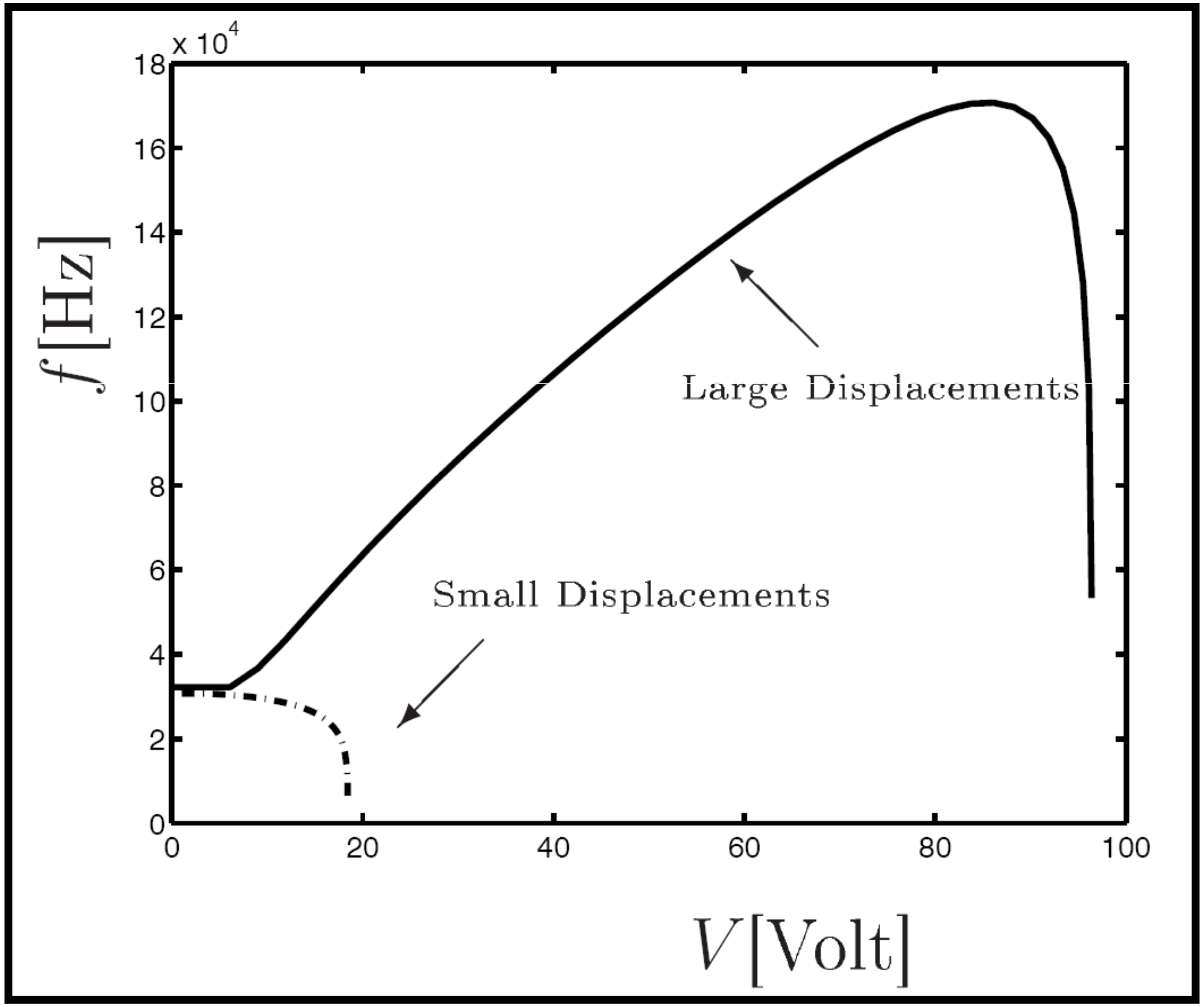
Application 1: Thin Beam, large gap

Static Equilibrium position



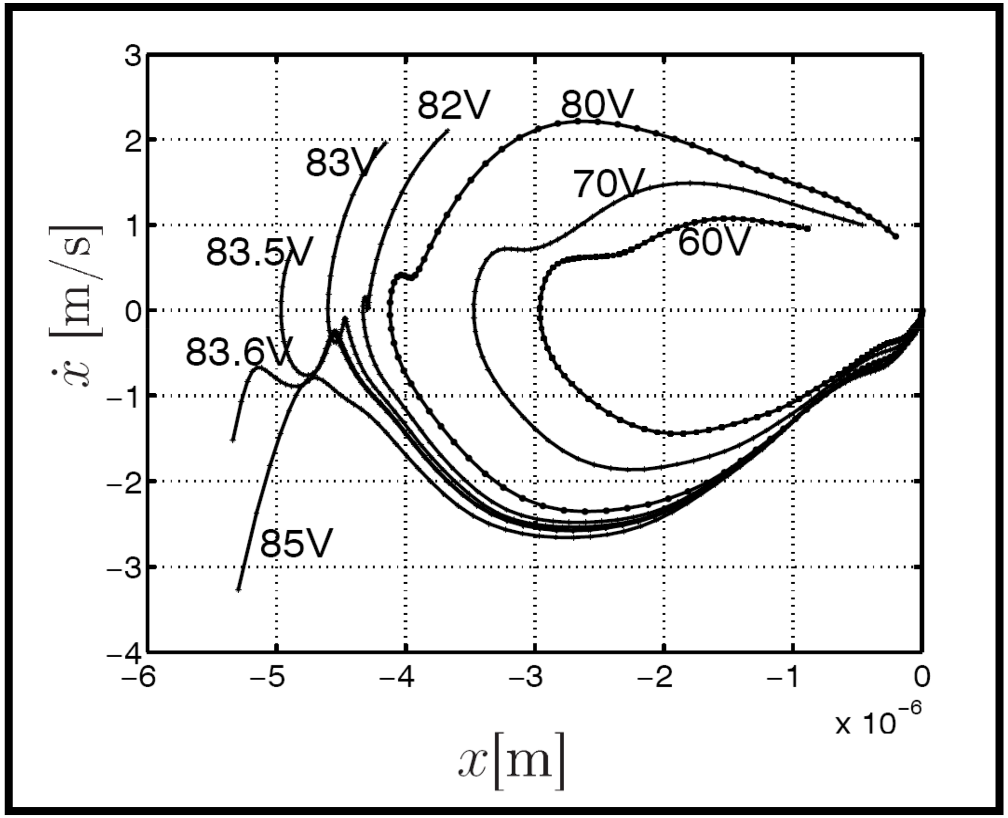
Application 1: Thin Beam, large gap

Natural frequency



Application 1: Thin beam, large gap

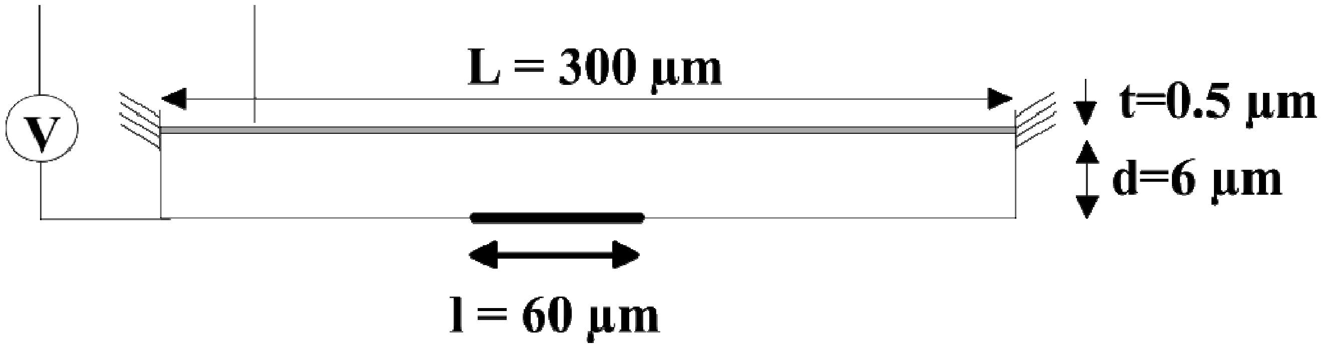
Dynamic Behaviour



Static Pull-in Voltage	96.4 V
Dynamic Pull-in Voltage	83.6 V
Difference	13%

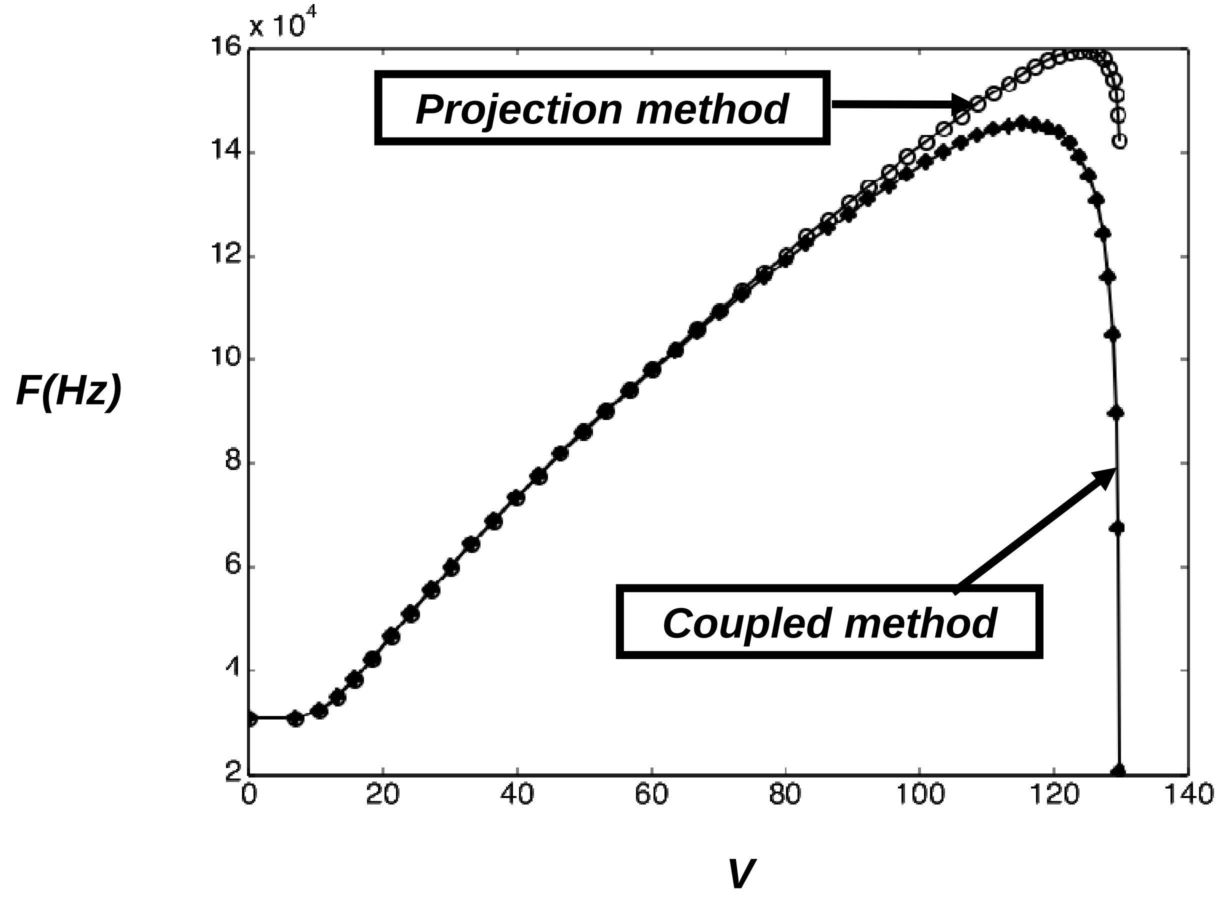
Application 1: Smaller electrode

- Modal solution :
  - Projection vs. Monolithical Method
    - Nearly same results for uniform electrodes
    - Difference when the lower electrode is reduced



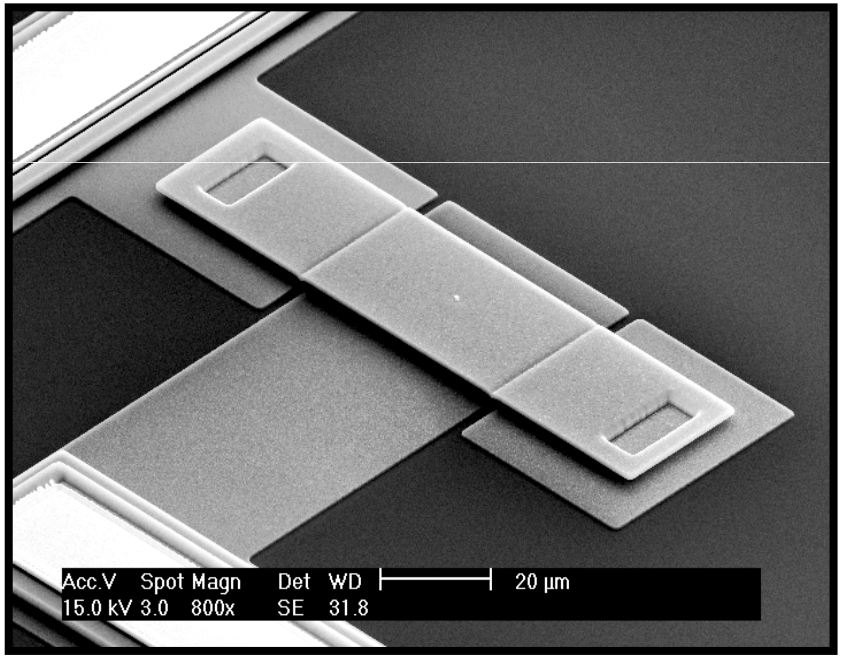
Application 1: Smaller electrode

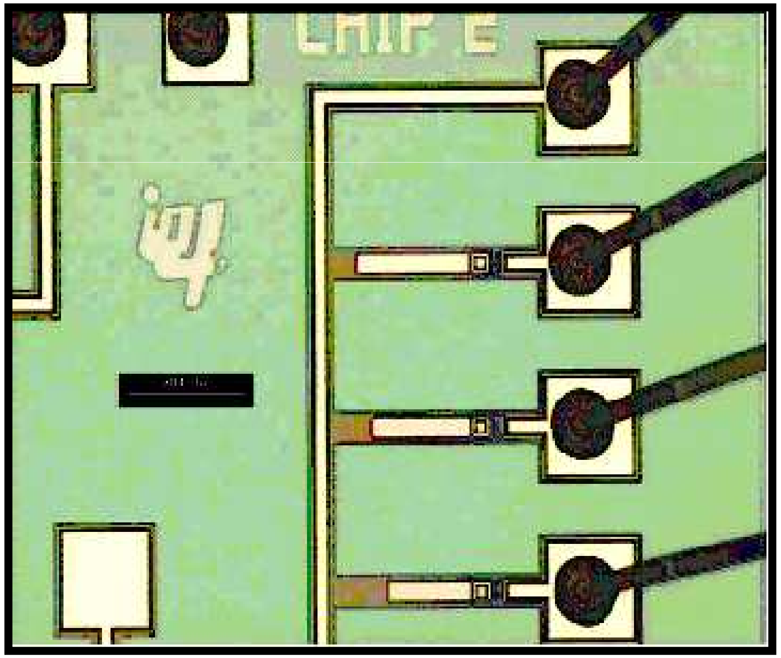
First Natural Frequency



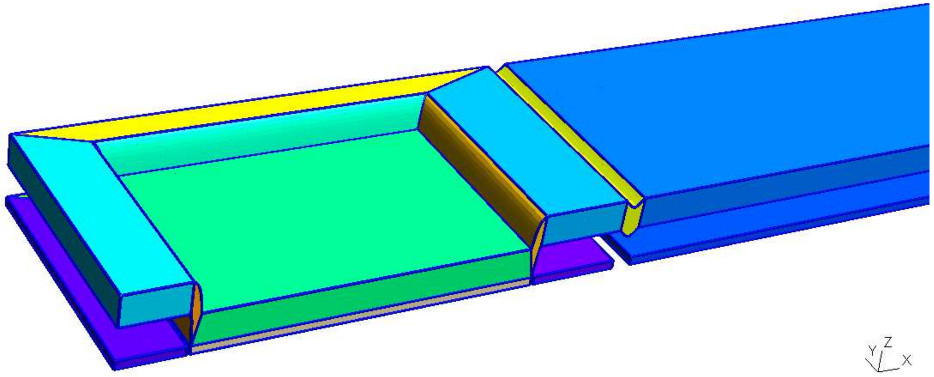
Application 2: Micro-resonators

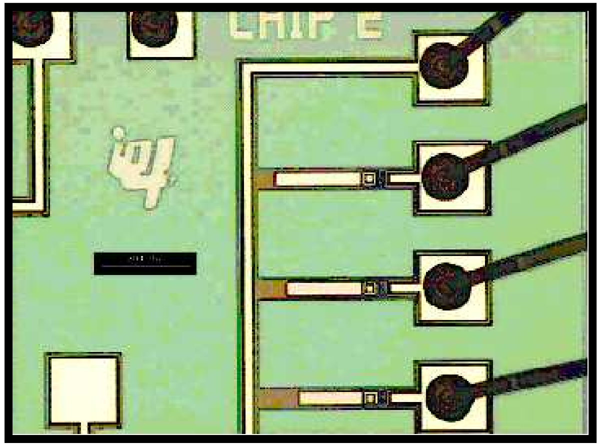
- **Studied Micro-devices**
  - **Electrostatically actuated micro-bridge (left)**
  - **Electrostatically actuated cantilever micro-beam (right)**

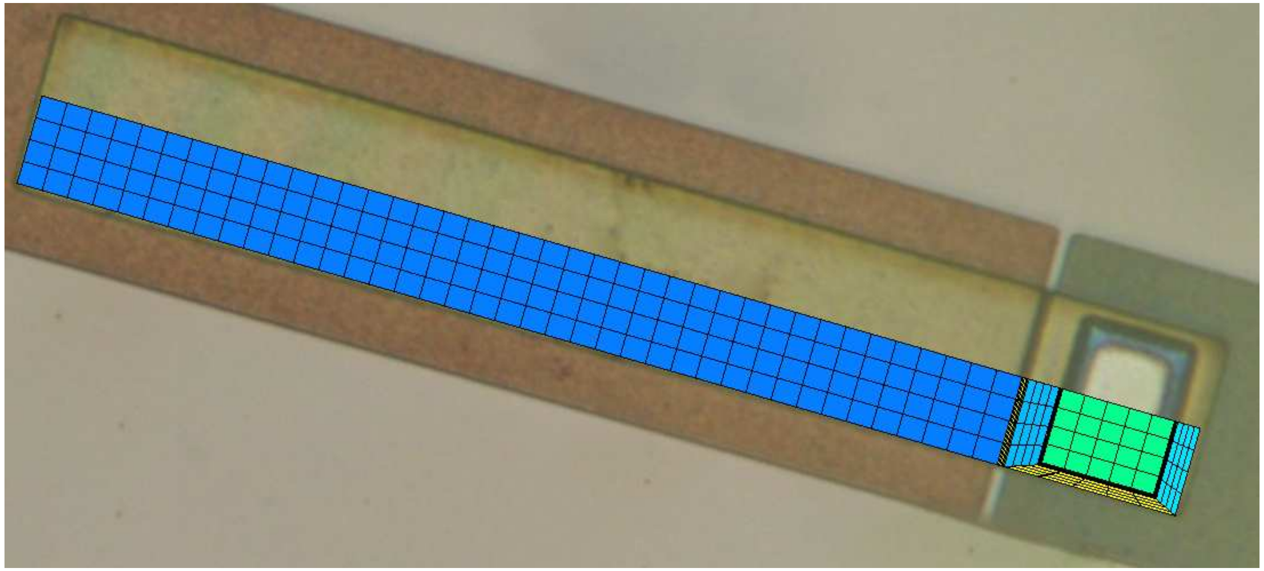




Application 2: Micro-resonators

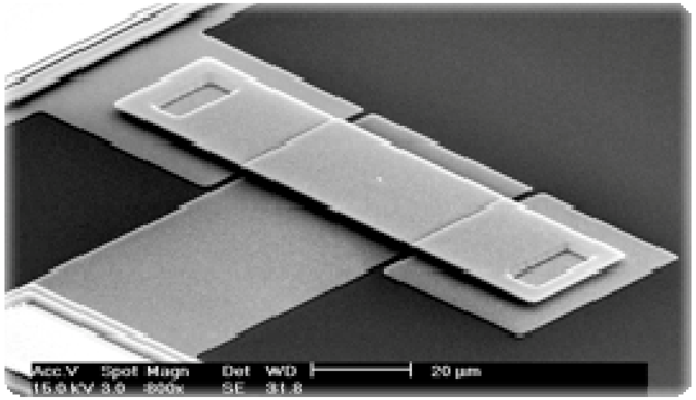




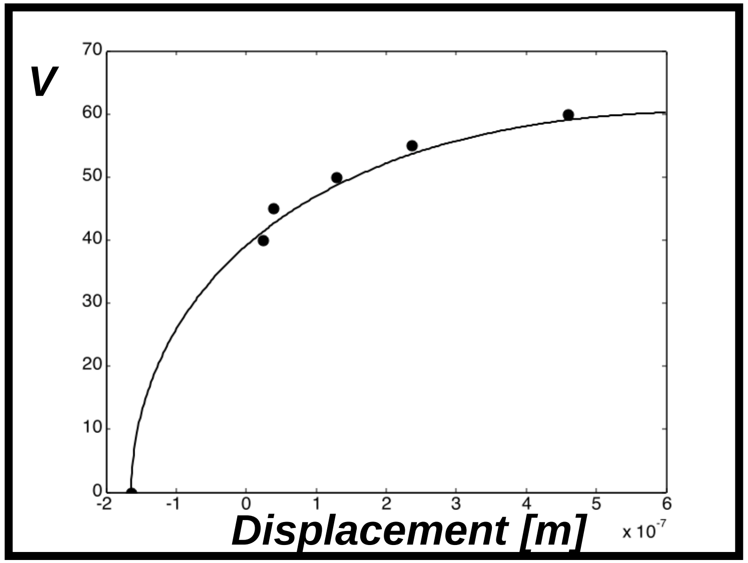


Application 2: Micro-resonators

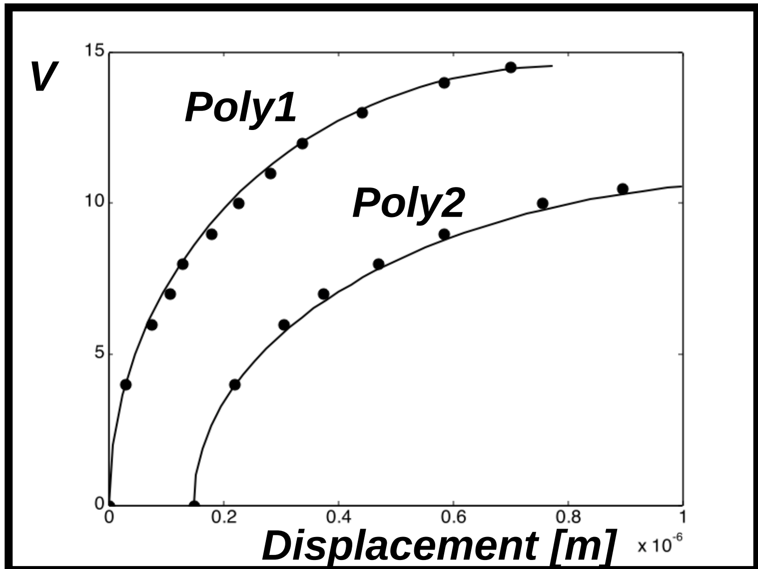
- Parameters to consider:
  - Pre-stress (due to manufacturing)
  - Shape of anchor
  - Model updating on E



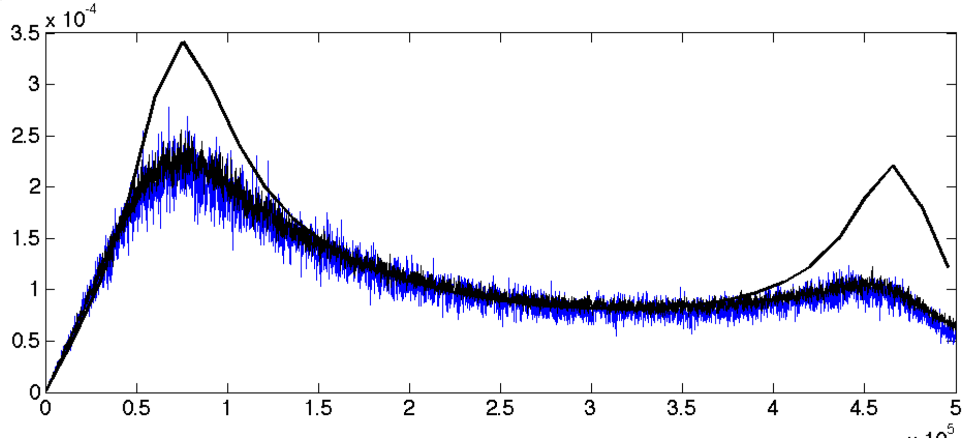
Micro-Bridge

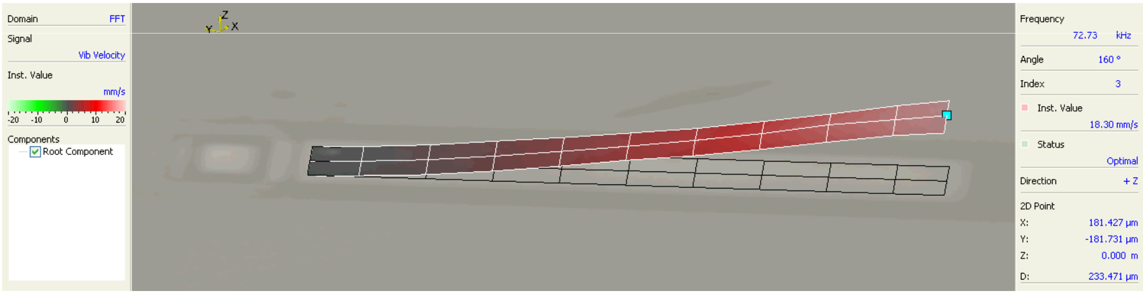


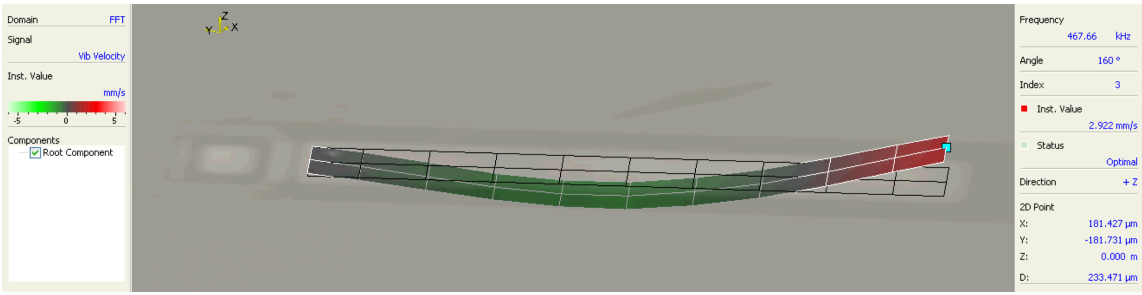
Cantilever Micro-Beams



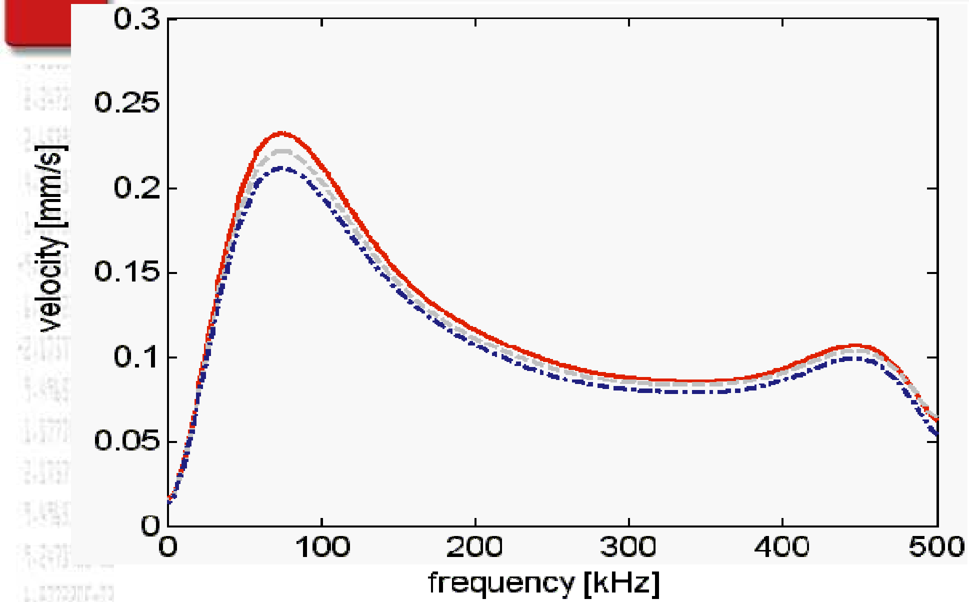
Application 2: Micro-resonators





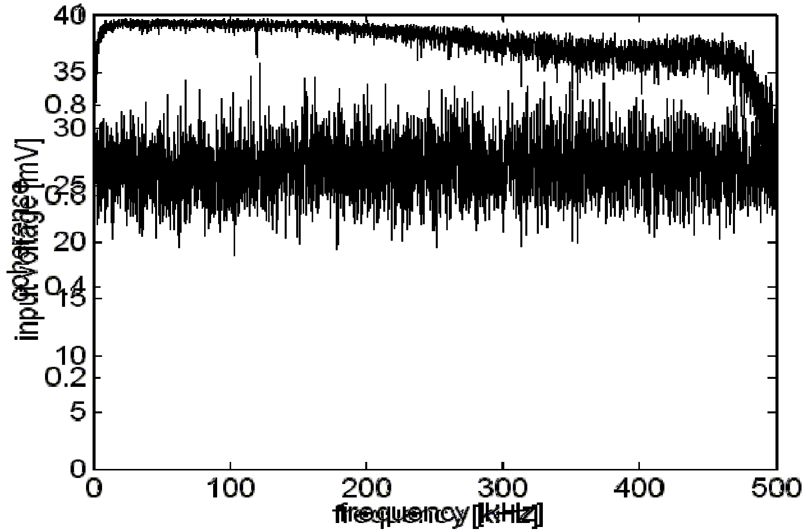


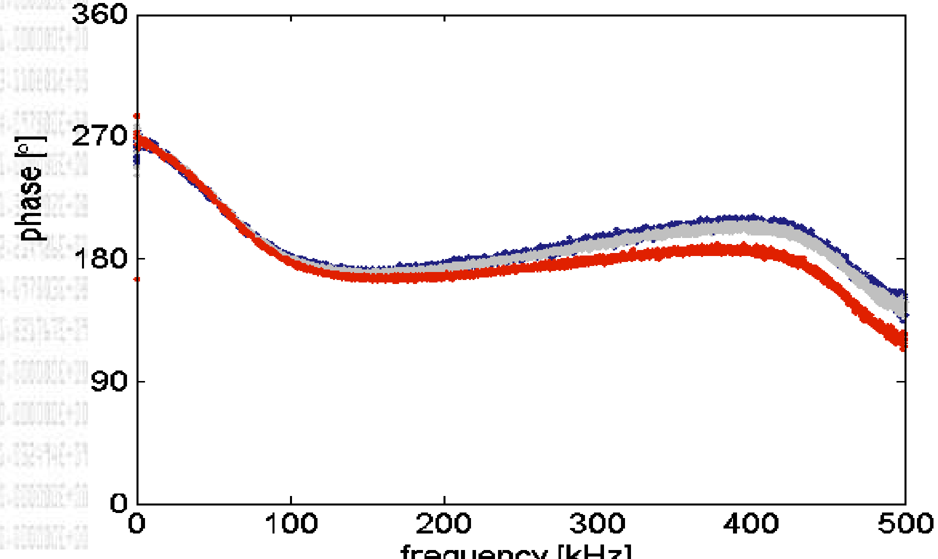
Application 2: Dynamic measurements in air

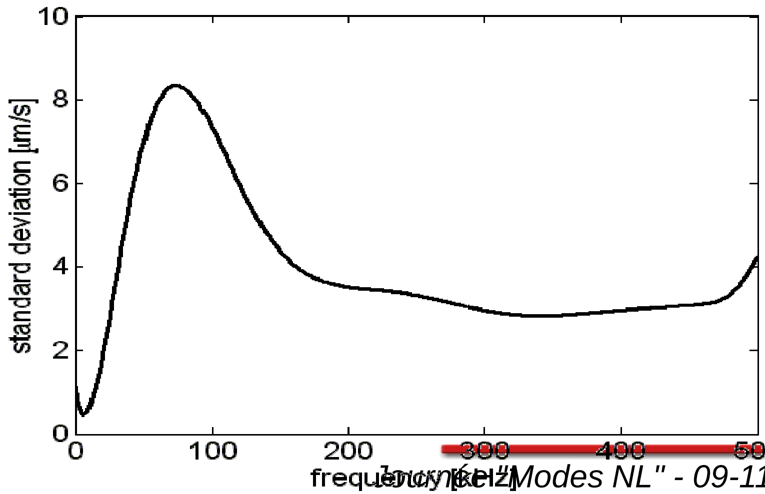


cantilever:

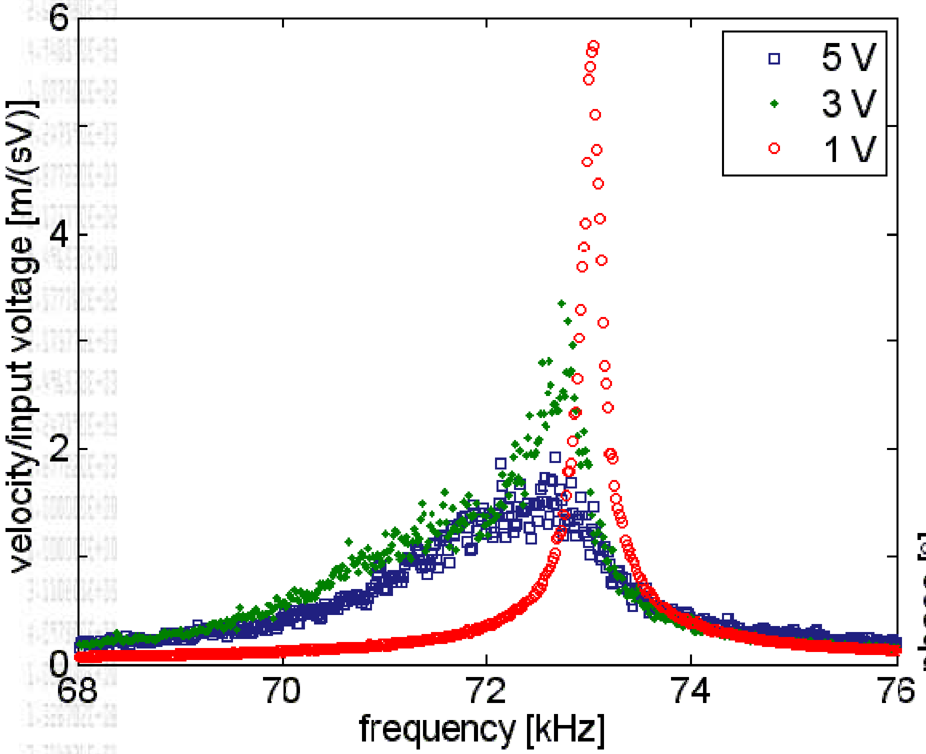
L=175 µm, W=30 µm, poly 1



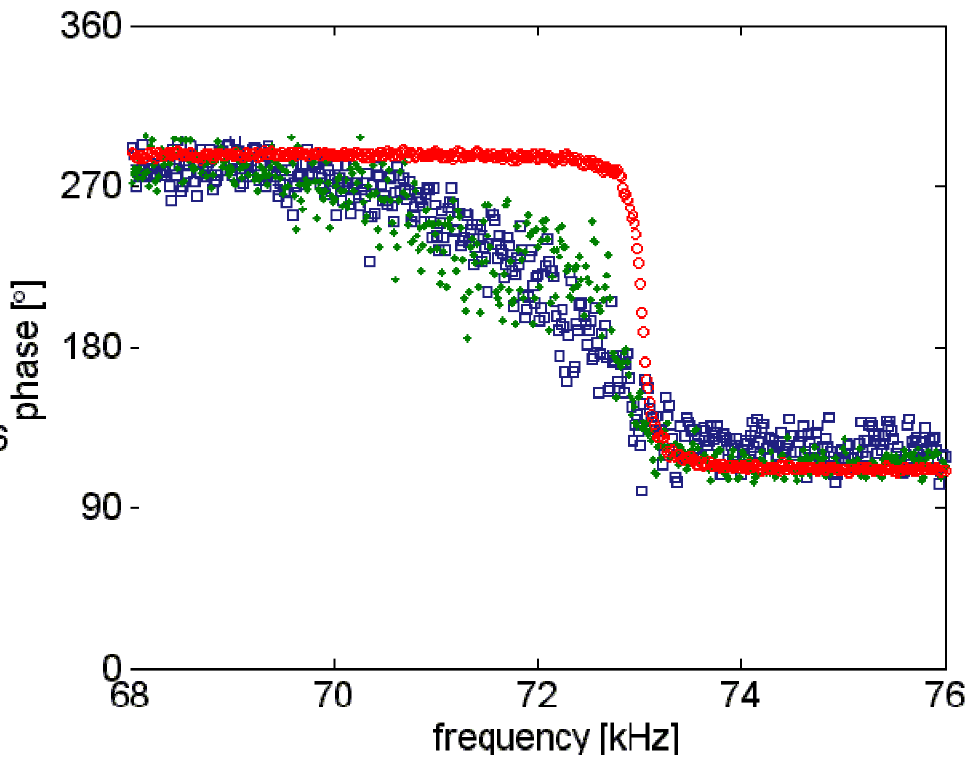




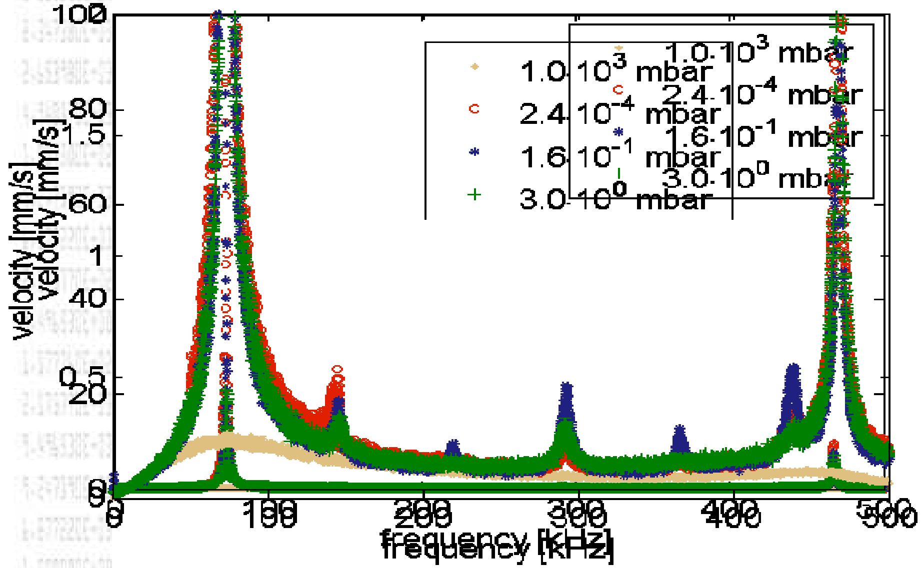
Application 2: Dynamic measurements in vacuum

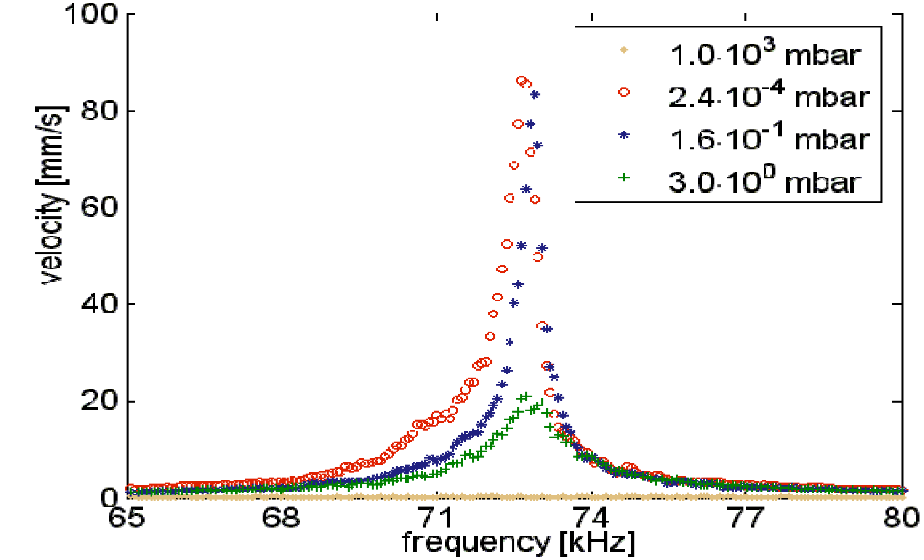


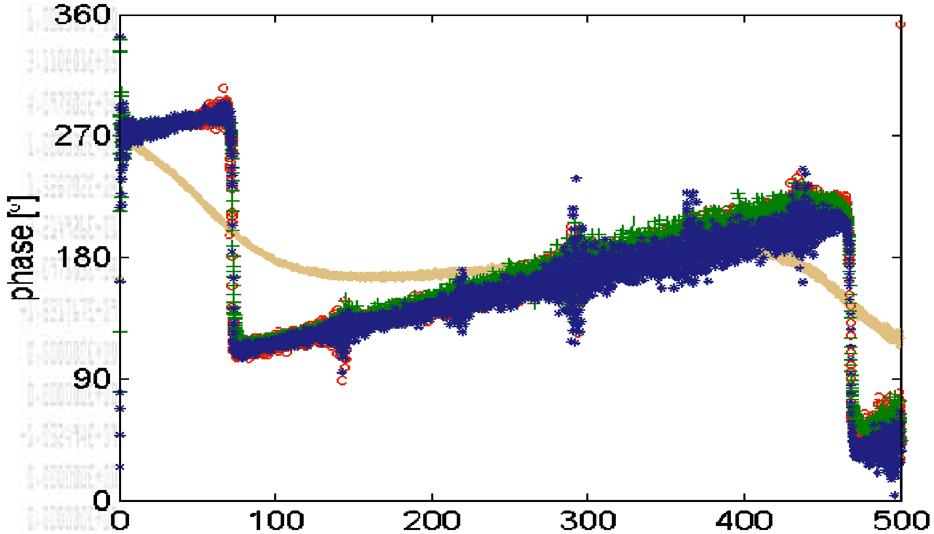
cantilever:
L=175 µm, W=30 µm, poly 1

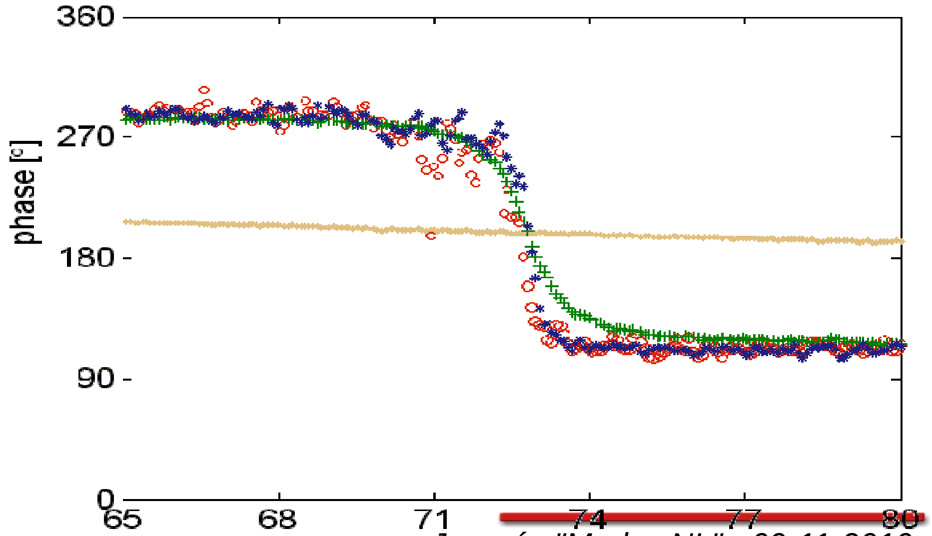


Application 2: Dynamic measurements in vacuum



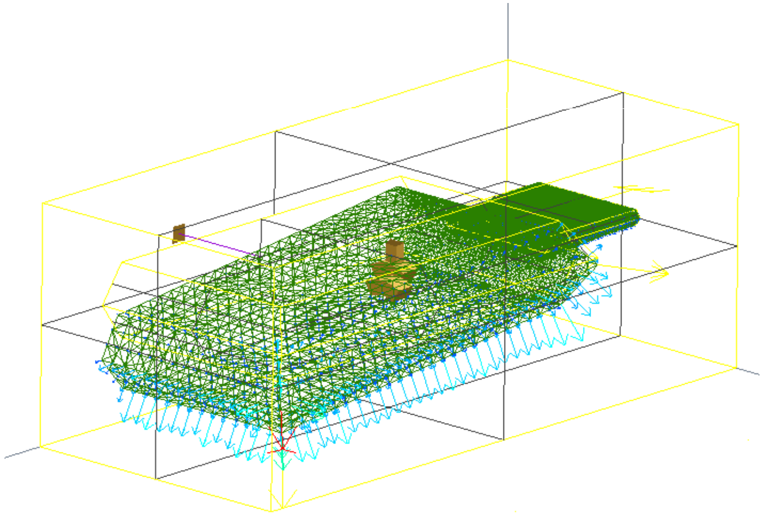




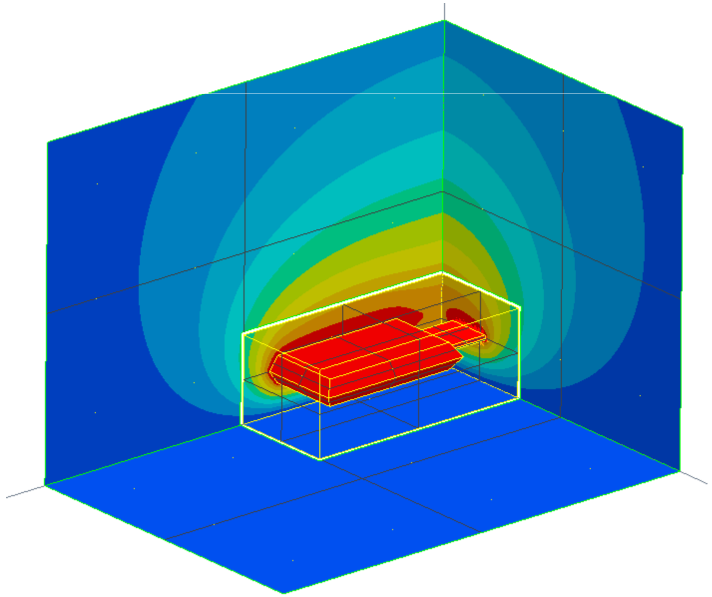


Integration in Oofelie::MEMS, driven by SF

- **Electrostatic effect added**
  - With BEM (no tangent stiffness)
  - With FEM (with tangent stiffness)
  - With FEM/BEM (with tangent stiffness)

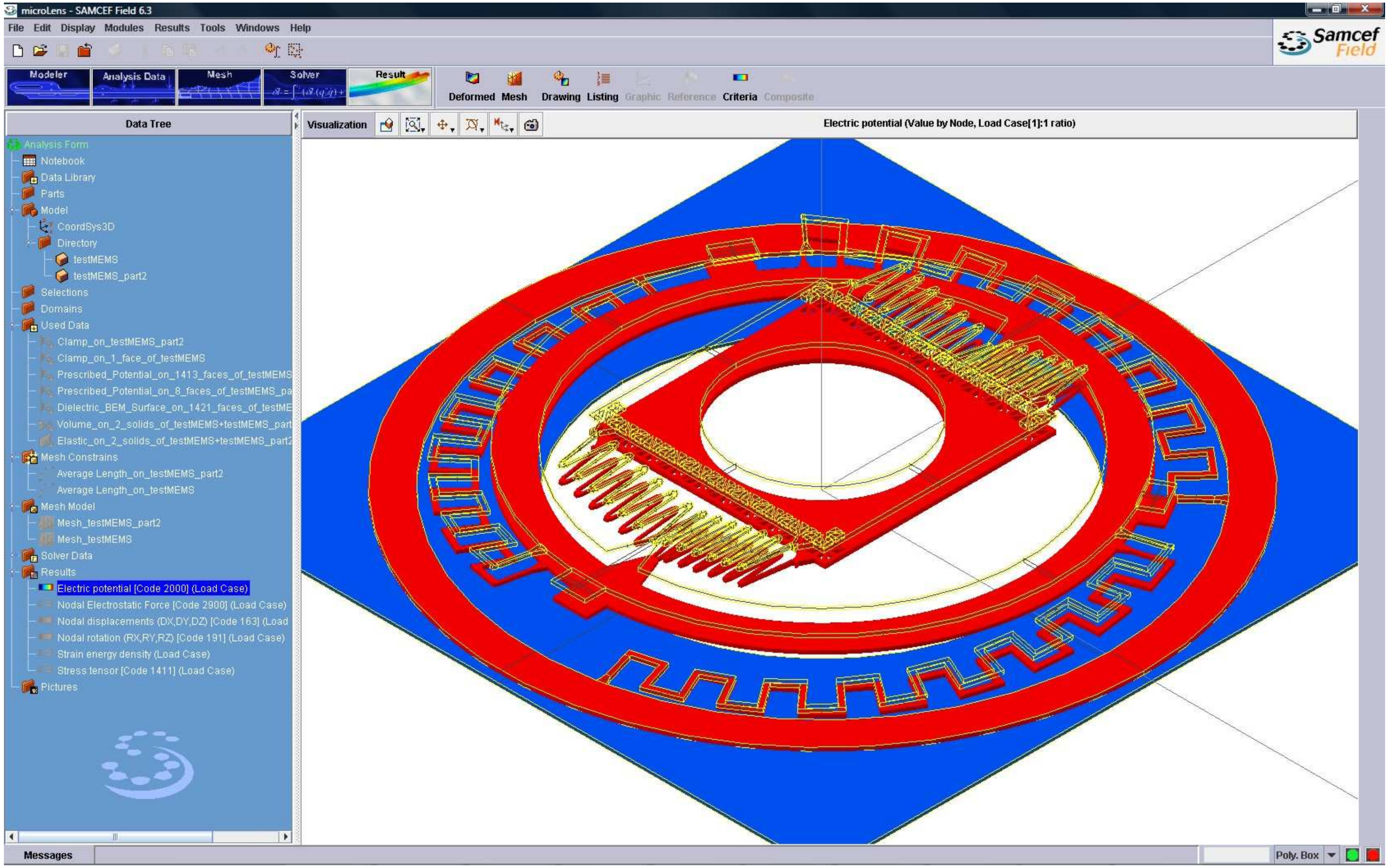


Structural displacements and electrostatic nodal forces



Electric potential distribution

Oofelie::MEMS, BEM using FMM



Electrostatically actuated micro-lens for biomedical application

(With courtesy of University of British of Columbia and British Columbia Cancer Research Centre, CANADA)

Perspectives

- Enhancement of damping modeling
  - Thermo-elastic damping (already available)
  - Fluid Molecular regime implementation
  - Support loss
  - ...
- Better definition of manufacturing pre-stress
- Introduction of a NL harmonic solver
- Extraction of tangent stiffness for BEM